



MODELS and METHODS for LOT SIZING and SCHEDULING

Carina Pimentel, Filipe Alvelos and J. M. Valério de Carvalho
Department of Production and Systems Engineering/Algoritmi Research Center
E-mail: carina@dps.uminho.pt, falvelos@dps.uminho.pt and vc@dps.uminho.pt

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ABSTRACT

The intent of this extended abstract is to briefly describe the work developed by the PhD student Carina Pimentel during her PhD studies, since 2007 until now. The core subject of this work is the development of models and methods for solving medium/short term production planning problems. Two different problems are studied. The first one is a classical lot sizing problem. For this problem several exact approaches were developed and compared. The second problem, motivated by a real situation, is a lot splitting and scheduling problem. For the latter, a new mixed integer programming model is developed, as well as several heuristic approaches that efficiently solve it.

INTRODUCTION

The main motivation of this work is the exploration of different approaches, based on mixed integer programming (MIP), on decomposition methods and on heuristics, for two production planning problems.

The first problem is usually associated with medium-term production planning decisions. It is a multi-item capacitated lot sizing problem with setup times (MICLESP). The problem consists in determining the quantity (or lot sizes) and timing of production for several final products over a number of finite production periods so as to satisfy a known demand for each product in each period, while respecting a known capacity in each period and minimizing the overall costs involved. There is an extensive body of research for this problem. The book by (Pochet and Wolsey 2006) is mandatory both for an introduction to the lot sizing literature or for a deep study. On it several related lot sizing problems are presented, including the problem studied in this work. Other review paper that presents a literature review for several lot sizing problems is the one from (Drexler and Kimms 1997). The main contribution of the work developed in this work for the MICLESP problem is the comparison of several different formulations, both in terms of lower bounds quality and in terms of integer solutions quality. Other contributions are the presentation of a general multiple Dantzig-Wolfe decomposition (MDWD) principle, the proposition of a

new model to the MICLESP based on MDWD, the presentation of a branch-and-price approach to solve this model and the presentation of a branch-and-price approach to solve the period decomposition model. This work is described in (Pimentel et al. 2010a).

The second problem studied in this work belongs to the short-term production planning and combines the lot sizing decisions with the scheduling decisions of a real problem, from a textile industry that produces fine knitted goods. The problem is related with the knitting production process in which the main components (garment parts) of a product are produced. The aim is to develop a tool to simultaneously solve two production planning problems of the knitting section: (1) a lot splitting problem in which the components demand is divided into several smaller size lots, to speed up production and (2) an assignment and scheduling problem, in which the lots determined in (1) are assigned and scheduled on a set of parallel machines. The literature about this problem is scarce, and none work combining all the characteristics of the problem studied in this work has ever been published, as far as the authors were able to find out. One innovative characteristic of the problem is the aim of minimizing the work-in-process levels between the knitting section and the following section. This characteristic creates dependencies between the products and the components. Typically in a lot sizing and/or scheduling problem this kind of dependencies does not exist. Another particularity of the problem we study, that was seldom studied in literature (Shim and Kim 2008), is the possibility to split the total demand associated with a given component into several lots of variable size, which can be produced simultaneously on more than one machine. For this second problem several contributions are given in this work. The more significant ones are the development of a new MIP model for a new integrated lot splitting and scheduling problem on parallel machines (LSSPPM), as long as the development and comparison (between each other and with the MIP model) of several local search based heuristics.

SOLUTION APPROACHES for the MICLESP

Because of the importance, both practical and theoretical, of the MICLESP, a lot of work is reported in the literature for this NP-hard problem. See for example (Karimi et al. 2003) for a review of solution methods (exact and heuristic ones).



We solve the MICLESP by exact methods, starting from a classical MIP model. Our first two approaches, the product and the period decompositions, are based on the application of the Dantzig-Wolfe decomposition principle (Dantzig and Wolfe 1960) to this classical model. They differ in the way the subproblems are defined. In the product decomposition, we leave the capacity constraints in the master problem, getting one single item uncapacitated lot sizing subproblem for each product. In the period decomposition, we leave the demand constraints in the master, getting one continuous knapsack subproblem with setups for each period. A third decomposition model results from the application of multiple Dantzig-Wolfe decomposition. This is a general decomposition methodology, where the emphasis is on obtaining good quality lower bounds. In our work, the motivation is to obtain optimal integer solutions. When applied to the MICLESP, both subproblem types (product and period) are integrated in a single model, whose linear relaxation provides (equal or) better lower bounds than the ones of the two single decompositions of the classical model. The three decompositions just mentioned are solved by branch-and-price (Barnhart et al. 1998), which consists in combining column generation with branch-and-bound.

SOLUTION APPROACHES for the LSSPPM

The problem LSSPPM is concerned with the development of a weekly production plan for a knitting section composed by a group of identical knitting machines, establishing the quantities to produce of each component (organized into one or several lots), where and when to produce them. The demand quantity and the due date of the products are known in advance. Each component has a variable unit production time and there is a compatibility matrix between machines and components. Besides, there are release dates associated to the machines. The objectives are (1) to minimize total tardiness and (2) to minimize total deviations occurred during production of each product. We formulate this NP-hard problem into a MIP model (Pimentel et al. 2010b) and solve it through a state-of-the-art software. In order to improve the problem efficiency we solve it too by approximate methods (Pimentel et al. 2010b; Pimentel et al. 2010c). Firstly, we determine an initial solution to the problem using the two following methods: (1) a network flow heuristic and (2) a list scheduling constructive heuristic. The two methods are then used as a starting solution for four local search algorithms. The neighborhoods of the four local search algorithms are based on ahead/backward insertion of lots. Taking into account the performance of the local search algorithms, we combine them through a systematic change of the four neighborhoods, giving rise to two metaheuristics that are adaptations of the basic

VNS algorithm and of the basic VND algorithm (Hansen and Mladenovic 2001). The computational experiments show that feasible solutions, that outperform the MIP ones, can be quickly found even for high machine utilizations.

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AUTHOR BIOGRAPHY



CARINA PIMENTEL is a Ph.D. student in Production and Systems Engineering at University of Minho, Portugal. Her research interests include the development of exact and heuristic methods for production planning and scheduling problems.